Curved Space and Quantum Weirdness

Confronting Einstein's Universe

*A challenging 1 day program that introduces talented year11-12 physics students to the concepts that underlie **Einsteinian** physics.*

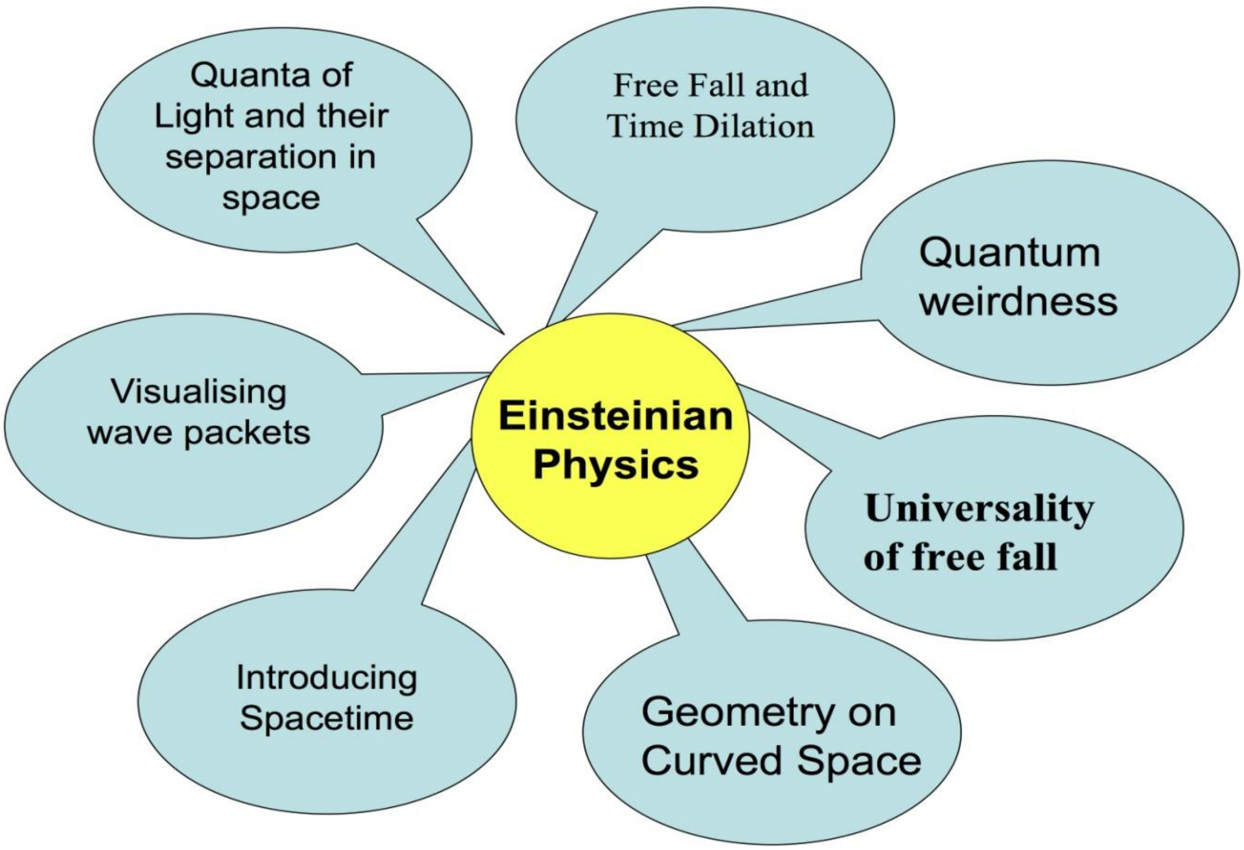
gravity DISCOVERY CENTRE

Introduction

The school physics curriculum is based mostly on **Newtonian** physics which itself is grounded in the 2300 year old geometry of Euclid. This program is designed to challenge you by introducing you to the concepts that underlie **Einsteinian** physics.

The key concepts of Einsteinian physics that are to be explored during this workshop are that:

a) light is a stream of photons that combine wavelike properties with particle like properties.
b) space-time is curved by matter and free fall trajectories trace the shortest path in space-time.



Some starting facts about waves: All waves carry energy and momentum. Waves can have arbitrary shape…they do not have to be sine waves. Normally, wave speed is fixed (like the speed of sound or the speed of light). Then waves add and subtract perfectly when they pass through each other. (In the shallow ocean this is not the case and waves can collide and bounce off each other.) The ability of waves to add up or cancel out is called *interference*. This is the key property of waves.

Waves can be polarised: they vibrate in a specific direction. When you measure light you find that it comes in little packets called photons. Similarly sound comes in little packets called phonons. *Exhibits at the GDC: the Wave Cable and the Interferometer- illustrate these ideas.*

Some starting facts about space: Space is not a rigid grid of imaginary lines. Space is curved by matter. The sun causes a one-part per million curvature of the space around it, but this has been measured to high precision. Our Lycra sheet allows you to visualise the process of space-time curvature. *Exhibits at the GDC: Einstein's Space, the Gravitational Lens and the Rubber Black Hole- illustrate these ideas.*

Group activity:

Lycrasheet experiment with whole class: observe matter making space curve, curvature making matter move, and see a 3-body interactions. **Students to take videos**

Small Group Activities:

Seven activities will be used to explore and explain the key concepts of Einsteinian physics. At the end we will confront the two key ideas:

a) **gravitational time dilation** that is a direct outcome of space-time curvature;
b) and the **quantum weirdness** that nobody really understands.

1. Intuitive visualisation of wave packet concepts

Wave Cable Experiments

a) Send wave packets down the giant wave cable: observe optical concepts: plane polarisation and circular polarisation, waves passing through each other.
b) Can you sense the momentum of an arriving wave packet?
c) Estimate the wave velocity.

Vortex Ring Experiments

Vortex rings are special confined motions of fluids that carry energy and momentum. In special fluids called superfluids they are also quantised into tiny packets of fluid circulation.

a) Make your own giant vortex ring.
b) Note that the vortex rings carry momentum: think of an experiment that could measure this momentum.
c) Can you observe a relationship between the ring size and the ring velocity? Make a rough plot of this relationship on a graph.

2. Space-time

a) Space-time diagrams in 2 dimensions: we can’t think in 4-dimensions so we make graphs with just one dimension of space and the other axis is time.

b) What scale do we use for the axes? Why SI units are are not acceptable in space-time?

c) What is the universal conversion factor between space and time?

Choosing space-like axes or time-like axes? Arbitrary choice. Time units: 1 meter = 1/300,000,000 of second or about 3 nanoseconds

Space units: 1 second= 300,000,000 meters

<u>Worksheet exercise 1:</u> use **$s=1/2gt^2$** to plot the space-time trajectory for a falling water balloon from the leaning tower (height 40 m).

a) How many seconds to hit the ground?

b) Plot the space-time trajectories, first in SI units (meters, seconds), then in Einsteinian units. Label the axes with rough, estimated numbers.

c) What dimensions of graph paper would be required to plot the space-time trajectory accurately to scale in units of meters?

<u>Worksheet exercise 2:</u>

You try to drop a pair of water balloons simultaneously from the Leaning Tower. What timing error in release time between two water balloons creates 5cm separation at the point of impact?

Hint: use s=40m and s=40.05m

3. Universality of Freefall

Einstein's theory of space, time and gravity, called the General Theory of Relativity is founded on the universality of free fall discovered by Galileo at the Leaning Tower of Pisa. Gravitational acceleration is the same for all bodies whatever their composition. This is called the Equivalence principle.

Read Galileo's claims at base of the leaning tower and be prepared to test his claim.

a) Water balloon experiments: *Confirm worksheet exercise 2 and read exercise 3 before you do the experiments.* Take some time in your small group to plan your water balloon experiments from the Leaning Tower. Consider the need for observers at the ground. If you are using photography you may need a trial run.

b) *Test for confounding variables, to find out if Galileo was telling the truth.*

Worksheet exercise 3:

a. Consider the universality of free fall discovered by Galileo. How serious are the confounding variables?
b. Plan and conduct an experiment that will test the two main confounding variables. Help needed? - *see last page*
c. Could Galileo have done the experiment or was it a thought experiment?
d. Aristotle stated that things fall at a speed proportional to their mass. Do you really need a Leaning Tower to prove Aristotle was wrong?

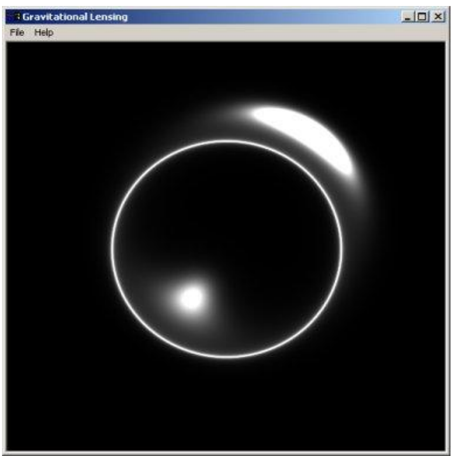
The two key confounding variables are a) Initial conditions: time of release and b) wind resistance. Is the effect of wind resistance greater for larger or smaller objects?

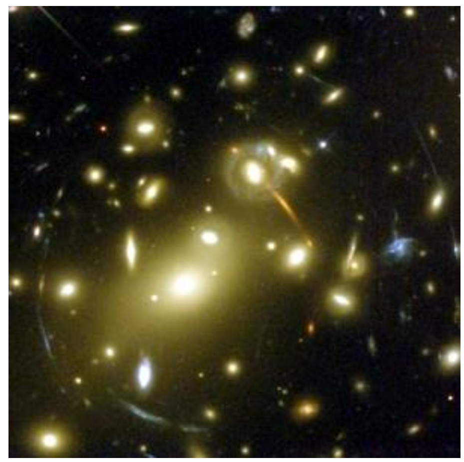
Your answers:

4. Experiments in Geometry on Curved Space

Basic Ideas: What do we mean by a straight line when space is curved? Surveying with poles is equivalent to surveying with laser beams or starlight. All methods use light paths to define straightness. We use toy cars to replace laser beams in 2-dimensional curved space. We will do experiments using the **spherical whiteboard**, the **rubber black hole** and the **gravitational lens.**

a) Use toy cars to draw various sized triangles on the spherical whiteboard. Use a pen and string to draw some concentric circles. Are the formulae you learnt at school correct?
b) Plot out parallel lines on both the spherical white board and the rubber black hole. Do they follow the high school definition of parallel lines?
c) Use toy cars to plot straight lines close-in to the rubber black hole.
d) Finish this component by photographing a friend through the GDC's **gravitational lens**: this lens simulates the real curved space geometry at true scale if the earth was compressed to a black hole.
e) The images below show the gravitational lensing distortions of galaxies that we see with modern telescopes: there are big distortions due to space being curved.





Photograph a friend through the gravitational lens: try to observe effects like the picture above.

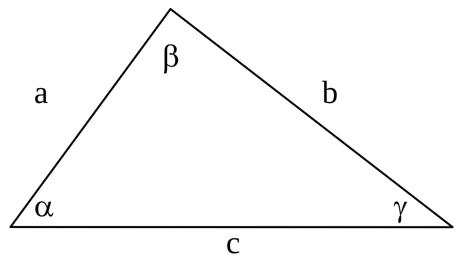
__Worksheet exercise 4:__

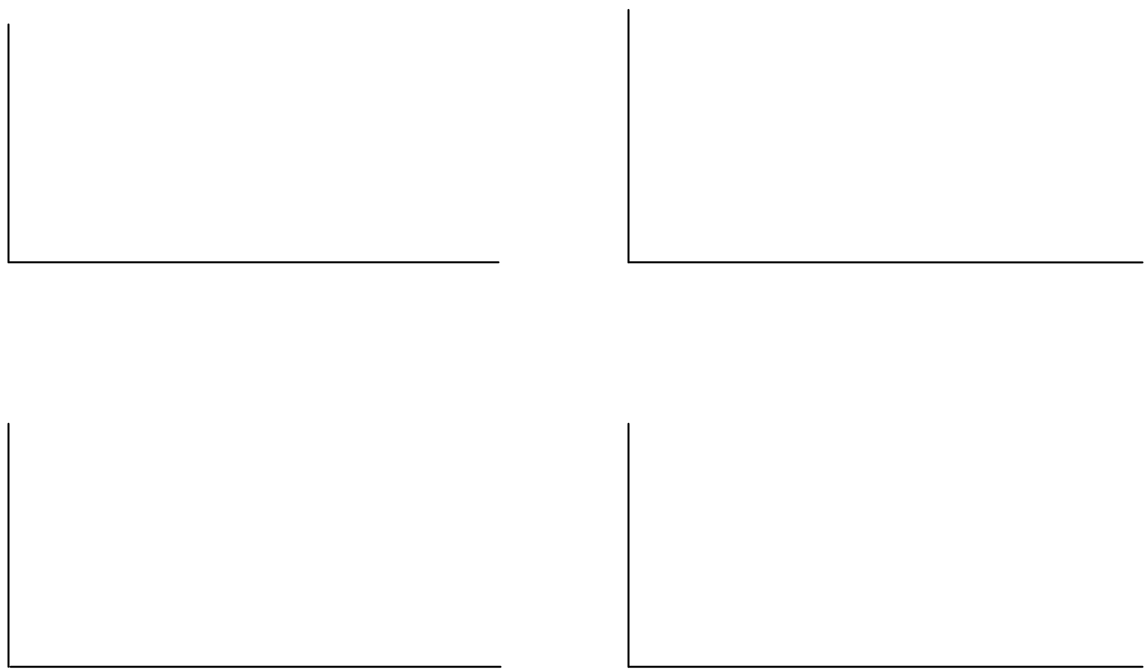
1. Make rough estimates of the errors in geometrical formulae that you have been taught at school for geometry on our spherical whiteboard: Sum of the angles of a triangle, area of a circle, Pythagoras's theorem, the perimeter of a circle. You can use tape measures and protractors or you can make more theoretical estimates.
In each case plot a graph of the fractional error in each formula as a function of size.
Example: For the formula Area= πr^2, you know that for all circles on flat paper Area/ πr^2 =1 exactly. So plot Area/ πr^2 as a function of circle radius, for circles drawn on the spherical whiteboard. For small radius the value is 1 but it becomes much larger as the radius increases.

Plot rough graphs for each of the four key geometric formulas, all as a function of diameter (for circles) or triangle maximum dimension:

a) Sum of the angles of a triangle, plot $(\alpha+\beta+\gamma)/180$
b) Area of a circle: plot Area/ πr^2
c) Pythagoras's theorem: plot $(a^2+b^2)/c^2$

d) Perimeter: plot perimeter/$2\pi r$





5. Quanta of light and their separation in space

Key idea: light comes in quanta called photons. The energy of a photon is hf where h is Planck's constant = 6.6×10^{-34} Joules per Hertz.

a) Optical power of monochromatic light: Power = N.hf where N = photons per second, hf = energy of quantum, h=Planck's Constant
b) Working with photons (whiteboard discussion): A zero magnitude star (near brightest, eg Vega) has a flux of about 10^6 photons/cm^2 /sec. A 6^{th} magnitude star (just visible to naked eye is 250 times less bright, 4000 photons/sec /cm^2. Through the 3mm aperture of your eye there are about 300 photons per second.

c) Interference of photons 1: Soap bubbles: observe and try to photograph optical interference in a soap bubble
d) Interference of photons 2: Observe interference fringes in the model gravitational wave detector.
e) Interference of photons 3: Pass laser pointer light through textile fibres. See fringes on the screen.

Worksheet exercise 5:

a) How far apart are the bullets from a machine gun if it fires 20 rounds per second at 1000 meters per second?

b) How many photons per second are emitted from a 1mW red laser pointer (630nm wavelength, E=hf = hc/(wavelength). How far apart are the photons on average?

c) How far apart are the photons that enter your eye from a 6^{th} magnitude star?

6. Quantum weirdness

a) Remember that photons travel at 3×10^8 meters per second. From *Worksheet exercise 5* we know that the photons are very far apart.
b) Thought experiment: suppose the light level was greatly reduced for your soap bubbles or the interferometer. If you took a long time exposure photo would you expect to see the same interference pattern?

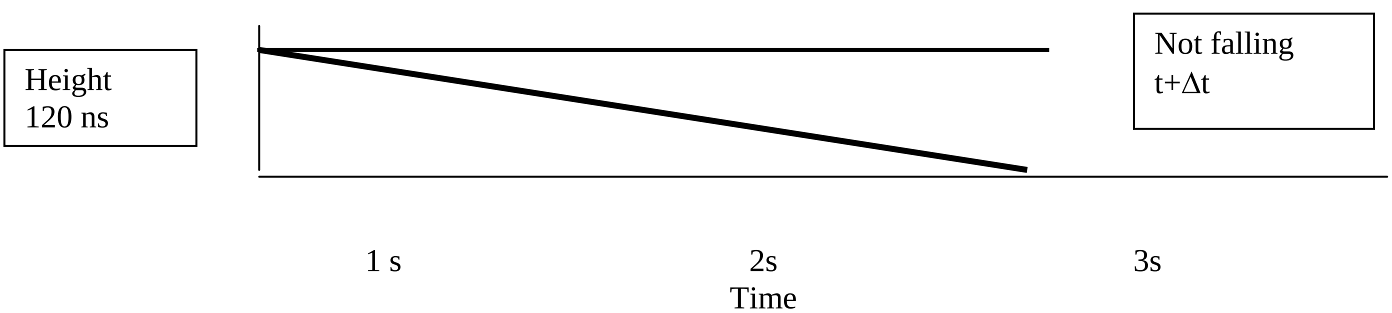
Information: *physicists have tested this very carefully and the interference pattern is always present however much you reduce the light intensity*

Conclusion: *photons interfere as if they are a continuous stream of waves, but they are not. This is weird. Nobody understands it. Physicists use the term 'wave function' to describe a wave of probability that determines the place where the photons will appear when you measure them.*

7. Free fall and Time Dilation

Information: Einstein says free fall trajectories are the shortest paths in space time (analogous to straight lines in flat space). All objects in free fall (like planets, asteroids, and astronauts in the space station) are following the shortest path in space-time. Whenever you do something to prevent free fall space-time trajectories are extended and, in particular, time is stretched. We will calculate this effect.

a) How can the free fall trajectory for the falling water balloon be shorter than the trajectory of not falling at all for the same amount of time?
b) _Worksheet Exercise 6_: Plot free fall and not-falling space-time trajectories, approximating the trajectory to a Pythagorean triangle.
c) How much stretching of the time axis would you need for the free fall trajectory (the hypotenuse) to be shorter than the non-falling trajectory?
d) If you lived on top of the Leaning Tower of Gingin for 1 year how much would your atomic clock be ahead of your friend's clock on the ground?



Conclusion: *So now you know that things fall not because of some mysterious force called gravity, but because time is slowed down when things are prevented from following their natural free fall trajectory.The stretching of the time axis with height is curvature of space-time.*

Interesting fact: *GPS receivers only allow you to tell you position on the earth because the atomic clocks in the GPS satellites are corrected for gravitational time dilation.*

Help page

Confounding variables
The two key confounding variables are a) Initial conditions: time of release and b) wind resistance. Is the effect of wind resistance greater for larger or smaller objects?

How to calculate the effect of gravity on time.

Approximate the free fall space-time trajectory of an object falling from the Leaning Tower as a hypotenuse of a triangle as shown in the diagram below.
The height using time units is $s/c = 1/2gt^2/c$
The time t to fall to the ground is the t in the above formula.

Now we consider two cases:
a) Falling: The distance in space-time includes the space distance and time distance.

By Pythagoras: $h^2 = \left(\frac{1}{2}\frac{g}{c}t^2\right)^2 + t^2$

b) Not falling: length of trajectory is t (no space distance, only time distance)

Key question: *How much do we need to stretch the time distance for t (not falling) to be as long as the hypotenuse?*

Let us suppose that time distance for the top of the tower has been stretched by Δt: the not falling trajectory now has a length of $t + \Delta t$. If Einstein is right this distance must be longer than the free fall distance.

Then, using Pythagoras, we must have: $h^2 = \left(\frac{1}{2}\frac{g}{c}t^2\right)^2 + t^2 < (t + \Delta t)^2$.

Now we just need to solve for Δt. Because this equation is an inequality, our solution will also be an inequality. (Think of < being an equals sign, solve in the usual way, then make sure you are sure whether to write the answer with > or <).

Remember Δt is tiny so Δt^2 is negligible. We can ignore this term and rearrange the result and divide by the total height so we have an answer for the time dilation per unit height.

The answer is $\Delta t > (¼)\, g/c^2$ per meter.

We made a big approximation because it is not a triangle. The true answer is $\Delta t = g/c^2$ per meter.

For the mathematically talented: Try calculating the time dilation using the proper parabolic shape of the trajectory. Note that we only calculated how much difference was required to make the free fall distance *equal* to not-falling when in reality the free fall trajectory is actually shorter…so the time dilation is actually 4 times the result we obtained.

Take Home Ideas

e) Interference is interference of possibilities, not photons!
f) An extension of these ideas explains why light travels in straight lines, and the laws of reflection. It is all quantum physics.
g) If anyone tells you they understand it they are either lying or stupid!
h) Things fall because that way they get old quickest.
i) If Newton had been born on an asteroid or in the spaceship he would never have discovered the law of gravity. Gravity is something we invented because we live in a very special place in the universe.
j) Anytime you do anything to stop things following free fall trajectories they take longer to get to their space-time destination…old age!
k) At the surface of a black hole time comes to an end!
l) Quantum Physics and Einstein's Theory of Gravity are incompatible: we may be able to see the breakdown when we can first observe gravitational waves from new-born black holes
m) *All these ideas come together in gravitational wave astronomy*.

Workspace for exercises: